Fractional projective-gap inequalities for harmonic Gaussian chaoses

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Abstract

Let F belong to a fixed Gaussian Wiener chaos, normalize $\mathbb{E}F^2 = 1$, and set $Z = |DF|^2/M$, where M is the chaos order. We study the fractional projective defect

$$\mathcal{K}_r(F) = \mathbb{E}[(F^2 - Z)Z^r], \qquad r > 0.$$

This compares the square of one chaos element with its normalized Carré-du-champ under a singular fractional weight. We prove three dimension-free or orbit-complete results. First, for every nonzero element of the second chaos, every $r > 0$, and every shift $\tau \geq 0$, the shifted defect $\mathbb{E}[(F^2 - Z)(Z + \tau)^r]$ is strictly positive. Second, the same shifted inequality holds in every degree for products of mutually orthogonal Gaussian linear forms. Third, at $r = 3/20$ we prove the defect for every homogeneous harmonic cubic in three variables. The last result is reduced to a signed resolvent family and closed by an exact rational Bernstein certificate on a three-parameter canonical orbit space; a short verifier regenerates all moments and inequalities from the cubic itself. We also give equivalent spherical and radial-energy formulations and explain why an ordinary variance estimate is unavailable at the nodal set. The results settle complete low-degree and all-degree split families; they do not assert the corresponding inequality for arbitrary chaos order.

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1 Introduction

Let γ_s be standard Gaussian measure on $\mathbb{R}^s$ and let

$$\mathcal{L} = \Delta - x \cdot \nabla$$

be the Ornstein–Uhlenbeck generator. If H is a homogeneous harmonic polynomial of degree M, Euler's identity and harmonicity give $\mathcal{L}H = -MH$; hence H is an element of the Mth Wiener chaos. Gaussian integration by parts then yields

$$\mathbb{E}|DH|^2 = M\mathbb{E}H^2.$$

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After the normalization $\mathbb{E}H^2 = 1$, the nonnegative random variable $Z = |DH|^2/M$ also has mean one. The equality of the first moments does not decide the sign of

$$\mathcal{K}_r(H) = \mathbb{E}[(H^2 - Z)Z^r], \tag{1}$$

because the two terms are integrated against different size-biased laws.

Moment inequalities on fixed Wiener chaoses, including product inequalities for squared chaos variables, have been developed using the Ornstein–Uhlenbeck semigroup and Malliavin calculus; see, for example, [3, 4, 2]. The comparison in (1) is of a different type: it couples one chaos element to its first derivative and uses a generally nonintegral power of the Carré-du-champ. At $r = 0$ it is an equality. At $r = 1$ an orthogonal-chaos decomposition gives a nonnegative sum, but interpolation between these two endpoints does not determine the sign at a fractional exponent.

Our results are as follows.

- In the second chaos we prove a strictly positive formula for every $r > 0$ and for the full shifted family $(Z + \tau)^r$.
- For $F = X_1 \cdots X_M$ we prove the same shifted inequality for all $M \geq 2$ and $r > 0$. Orthogonal invariance gives the corresponding result for products of mutually orthogonal linear forms.
- For every real spherical harmonic cubic on $\mathbb{S}^2$ we prove the first fully mixed cubic inequality, at $r = 3/20$. This is an exact computer-assisted theorem: the finite certificate uses rational arithmetic only, and its source code is supplied.

The exponent $3/20$ is the degree-three instance of

$$r_M = \frac{2M - 3}{2(M-1)(M^2 - M - 1)}. \tag{2}$$

This family occurs in the projective finite-dimensional obstruction in a nonlinear cone asymptotic. The present paper isolates and proves the standalone harmonic inequalities. It does not claim a solution of the full nonlinear Kondrat'ev program, and it leaves the arbitrary-degree version of (1) open.

Separable p-harmonic functions in cones lead to nonlinear spherical eigenvalue problems and homogeneity exponents; see [5, 6]. These works provide the analytic cone background, but do not contain the derivative-weighted Gaussian comparison studied here. Conversely, the inequalities proved below are only one finite-dimensional ingredient and do not supply the compactness and classification theory required for a nonlinear cone expansion.

Section 2 records the Gaussian, spherical, and variational forms. Sections 3 and 4 prove the two analytic families. Sections 5–6 give the cubic reduction and its exact certificate. The final section records the precise boundary of the results.

2 Three equivalent forms

Let Y be a degree-M spherical harmonic on $\mathbb{S}^{s-1}$ and let $H(\rho\omega) = \rho^M Y(\omega)$. Put

$$G = M^2 Y^2 + |\nabla_{\mathbb{S}} Y|^2, \qquad q = 2 + 2r, \qquad \delta = s + q(M-1).$$

If $X = R\Omega$ is a standard Gaussian vector, then R and Ω are independent, Ω is uniformly distributed on the sphere, and

$$H(X)^2 = R^{2M} Y^2, \qquad |DH(X)|^2 = R^{2M-2} G.$$

The chi-square recurrence $\mathbb{E}R^{a+2} = (s + a)\mathbb{E}R^a$ gives the following identity. Spherical integrals below use normalized surface measure.

Proposition 2.1 (Gaussian–spherical reduction). *For every $r > 0$,*

$$\mathcal{K}_r(H) = c_{M,s,r} \left\{ M\delta \int_{\mathbb{S}^{s-1}} Y^2 G^r - \int_{\mathbb{S}^{s-1}} G^{r+1} \right\}, \tag{3}$$

where

$$c_{M,s,r} = M^{-r-1} \mathbb{E} R^{(2M-2)(r+1)} > 0.$$

Proof. The second term in (1) has radial exponent $a = (2M-2)(r+1)$ and the first has exponent $a+2$. Since $s+a = s+(M-1)(2+2r) = \delta$, factoring the common radial moment proves (3). □

There is also a direct variational interpretation. For a fixed boundary trace Y, consider $u_\kappa(\rho, \omega) = \rho^\kappa Y(\omega)$ in the unit ball and its q-energy

$$\mathcal{E}_q(\kappa; Y) = \frac{\int_{\mathbb{S}^{s-1}} (\kappa^2 Y^2 + |\nabla_{\mathbb{S}} Y|^2)^{q/2}}{s + q(\kappa - 1)}. \tag{4}$$

Proposition 2.2 (Radial-energy derivative). *At $\kappa = M$,*

$$\partial_\kappa \mathcal{E}_q(M; Y) = \frac{q}{\delta^2} \left\{ M\delta \int Y^2 G^r - \int G^{r+1} \right\}. \tag{5}$$

Thus $\mathcal{K}_r(H) > 0$ if and only if decreasing the radial exponent from M is a strict q-energy descent direction.

Proof. Differentiate the numerator and denominator of (4). At $\kappa = M$ the numerator derivative is $qM \int Y^2 G^r$ and the denominator is δ. This gives (5), and Proposition 2.1 gives the last assertion. □

3 The complete second-chaos inequality

The next theorem does not require the polynomial to be homogeneous and harmonic; it holds for every finite-dimensional element of the second Wiener chaos.

Theorem 3.1 (Shifted second-chaos gap). *Let $F \not\equiv 0$ belong to the second Wiener chaos, put*

$$g = DF, \qquad A = D^2 F, \qquad Z = \frac{|g|^2}{2}, \qquad T = A[g, g].$$

Then, for every $r > 0$ and $\tau \geq 0$,

$$K_{r,\tau}(F) := \mathbb{E}[(F^2 - Z)(Z + \tau)^r] > 0. \tag{6}$$

More precisely,

$$K_{r,\tau}(F) = \frac{r}{2} \mathbb{E}(Z + \tau)^{r-2} \left((Z + \tau)|Ag|^2 + \frac{r-1}{2} T^2 \right). \tag{7}$$

Proof. Since $\mathcal{L}F = -2F$, Gaussian integration by parts gives, for smooth ϕ,

$$\mathbb{E}[(F^2 - Z)\phi(Z)] = \frac{1}{2} \mathbb{E}[F\phi'(Z)T].$$

The matrix A is constant, $DZ = Ag$, $DT = 2A^2 g$, and $g \cdot DT = 2|Ag|^2$. A second integration by parts, with $\phi(z) = (z + \tau)^r$, gives (7).

If $r \geq 1$, its integrand is nonnegative and nonzero on a set of positive measure. If $0 < r < 1$, Cauchy–Schwarz gives

$$T^2 \leq |g|^2 |Ag|^2 = 2Z|Ag|^2,$$

so the bracket in (7) is bounded below by $(rZ + \tau)|Ag|^2$. This is strictly positive on a set of positive measure. For $\tau = 0$, replace Z by $Z + \varepsilon$ and pass to the limit. Indeed $|Ag|^2 \leq CZ$ and $T^2 \leq CZ^2$, so the absolute value of each regularized term is bounded by a constant multiple of Z^r. □

At $r = r_2 = 1/2$, Theorem 3.1 and Proposition 2.1 prove the resonant spherical inequality for every quadratic spherical harmonic in every dimension. There is also a purely spherical matrix certificate, but (7) is both shorter and stronger.

4 Split harmonics in every degree

We next prove a full shifted theorem for a nontrivial family in every degree. It shows in particular that high degree alone does not obstruct the projective gap.

Theorem 4.1 (Products of orthogonal linear forms)**.** *Let $X_1, \dots, X_M$ be independent standard Gaussian variables, $M \geq 2$, and set*

$$F = \prod_{i=1}^{M} X_i, \qquad Z = \frac{|DF|^2}{M}.$$

Then for every $r > 0$ and $\tau \geq 0$,

$$\mathbb{E}[(F^2 - Z)(Z + \tau)^r] > 0. \tag{8}$$

The same is true after an orthogonal change of variables and after adding inactive Gaussian coordinates.

Proof. Write

$$A_i = X_i^2, \qquad P = \prod_i A_i, \qquad Q_i = \prod_{j \neq i} A_j.$$

Then $F^2 = P$ and $Z = M^{-1} \sum_i Q_i$. Exchangeability gives

$$\mathbb{E}[Z(Z+\tau)^r] = \mathbb{E}[Q_M (Z+\tau)^r].$$

Condition on $A_1, \dots, A_{M-1}$ and put

$$Q = Q_M, \qquad D = \sum_{i<M} \prod_{\substack{j<M \\ j \neq i}} A_j.$$

Almost surely $Q, D > 0$ and $MZ = Q + DA_M$. The conditional contribution to (8) is therefore

$$Q\mathbb{E}_{A_M}\left[(A_M - 1)\left(\tau + \frac{Q + DA_M}{M}\right)^r\right].$$

Because $\mathbb{E}A_M = 1$, the inner expectation is $\operatorname{Cov}(A_M, h(A_M))$, where $h(a) = (\tau + (Q + Da)/M)^r$ is strictly increasing. If A'_M is an independent copy, then

$$2\operatorname{Cov}(A_M, h(A_M)) = \mathbb{E}[(A_M - A'_M)(h(A_M) - h(A'_M))] > 0.$$

This proves the result. Orthogonal invariance of Gaussian measure and of the gradient norm gives the last statement. □

For $0 < r < 1$ there is a second proof, useful for comparison. Under the probability obtained by tilting Gaussian measure by F^{2+2r}, the variables $A_i = X_i^2$ are independent χ^2_{3+2r} variables. Thus, with $W_i = A_i^{-1}$ and $R = M^{-1}\sum_i W_i$, the sign of the unshifted defect is the sign of $\mathbb{E}R^r - \mathbb{E}R^{r+1}$. Strict concavity and convexity give

$$\mathbb{E}R^r > \mathbb{E}W_1^r, \qquad \mathbb{E}R^{r+1} < \mathbb{E}W_1^{r+1},$$

whereas

$$\mathbb{E}W_1^t = 2^{-t}\frac{\Gamma(3/2+r-t)}{\Gamma(3/2+r)}$$

implies $\mathbb{E}W_1^r = \mathbb{E}W_1^{r+1}$. This recovers strict positivity.

5 The fully mixed cubic reduction

Let $Y \not\equiv 0$ be a real spherical harmonic of degree three on $\mathbb{S}^2$, and put

$$B_Y = |\nabla_{\mathbb{S}} Y|^2, \qquad G = 9Y^2 + B_Y.$$

For $M = s = 3$ and $r = 3/20$, one has $q = 23/10$ and $\delta = s + q(M-1) = 38/5$. Proposition 2.1 shows that the Gaussian inequality is equivalent to

$$\int_{\mathbb{S}^2} G^{3/20}\left(\frac{114}{5}Y^2 - G\right) d\sigma > 0, \tag{9}$$

or, equivalently,

$$\int_{\mathbb{S}^2} G^{3/20}\left(\frac{69}{5}Y^2 - B_Y\right) d\sigma > 0.$$

5.1 A compact canonical chart

Scale and rotate Y so that its absolute maximum equals one and is attained at the north pole. Criticality removes the two $m = 1$ spherical harmonic coefficients. A rotation in the tangent plane diagonalizes the tangential Hessian. Every rotation-and-scale orbit consequently has a representative

$$\begin{aligned} Y = \frac{1}{2}\{2z^3 - 3z(x^2+y^2)\} + Bz(x^2-y^2) \\ + C(x^3 - 3xy^2) + D(3x^2y - y^3). \end{aligned} \tag{10}$$

At the pole the tangential Hessian has eigenvalues $-6 \pm 2B$, while on the equator the last two terms equal $C\cos 3\varphi + D\sin 3\varphi$. Hence every required orbit enters the relaxed compact set

$$|B| \le 3, \qquad C^2 + D^2 \le 1. \tag{11}$$

We shall certify (9) on this larger set; no unverified condition characterizing the maximum is used.

5.2 A signed resolvent family

For $0 < r < 1$,

$$x^r = \frac{\sin(\pi r)}{\pi}\int_0^\infty \frac{x}{x+t}t^{r-1}\,dt.$$

Thus it suffices to prove, for every $t \geq 0$,

$$R_Y(t) := \int_{\mathbb{S}^2} \frac{G}{G+t} \left(\frac{69}{5} Y^2 - B_Y \right) d\sigma > 0. \tag{12}$$

Let

$$U_k = \int Y^2 G^k \quad (0 \leq k \leq 4), \qquad V_1 = \int G^2.$$

Since $-\Delta_{\mathbb{S}} Y = 12Y$, one has $\int G = 21U_0$. Put $m_k = U_k/U_0$ and $e = V_1/(21U_0)$. With $h_t(x) = x/(x+t)$ and the probability measures proportional to $Y^2 d\sigma$ and $G d\sigma$, Jensen's inequality and a two-dimensional Cauchy–Schwarz trial give

$$\mathbb{E}_G h_t(G) \leq \frac{e}{e+t}, \qquad \mathbb{E}_{Y^2} h_t(G) \geq v^T M(t)^{-1} v, \tag{13}$$

where

$$v = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}, \qquad M(t) = \begin{pmatrix} m_2 + tm_1 & m_3 + tm_2 \\ m_3 + tm_2 & m_4 + tm_3 \end{pmatrix}.$$

Indeed the second inequality follows by applying Cauchy–Schwarz to the two functions

$$\sqrt{G/(G+t)} \quad \text{and} \quad \sqrt{G(G+t)}(a+bG),$$

and optimizing over (a, b). Consequently (12) follows from

$$\frac{38}{35} v^T M(t)^{-1} v > \frac{e}{e+t}. \tag{14}$$

After clearing the positive moment determinant, (14) is one quadratic in t. It is convenient to use unnormalized moments and define

$$\begin{aligned} &P_0 = U_1^2 U_4 - 2U_1U_2U_3 + U_2^3, && P_1 = U_1^2 U_3 - U_1 U_2^2, \\ &D_0 = U_2U_4 - U_3^2, && D_1 = U_1U_4 - U_2U_3, \\ &D_2 = U_1U_3 - U_2^2. \end{aligned}$$

Set

$$\begin{aligned} R_0 &= V_1(38P_0 - 35U_0D_0), \\ R_1 &= 21 \cdot 38U_0P_0 + 38V_1P_1 - 35V_1U_0D_1, \\ R_2 &= 21 \cdot 38P_1 - 35V_1D_2. \end{aligned} \tag{15}$$

The exact sufficient test is

$$R_0 > 0, \qquad R_2 > 0, \qquad R_1 \geq 0 \quad \text{or} \quad \Delta := 4U_0R_0R_2 - R_1^2 > 0. \tag{16}$$

The identity

$$R_2 = 35D_2 \left(\frac{114}{5} U_1 - V_1 \right) \tag{17}$$

will also verify that the moment matrix in (13) is positive definite.

6 The exact rational certificate

All sphere moments in (15) are rational polynomials in B, C, D. Direct reduction gives the invariant generators

$$a = B^2, \qquad s = C^2 + D^2, \qquad w = B^3(C^2 - D^2). \tag{18}$$

The substitution

$$a = b^2, \qquad w = b^3 su, \qquad 0 \le b \le 3, \quad 0 \le s \le 1, \quad -1 \le u \le 1 \tag{19}$$

covers every triple in (11). The verifier reconstructs every $U_0, \ldots, U_4, V_1$ from (10), reduces it to (18), and checks the substitution back into the original polynomial exactly.

We use the tensor-product Bernstein basis after the affine changes $b = 3\xi$, $s = \eta$, and $u = -1 + 2\zeta$. A polynomial is strictly positive on a box if every Bernstein control coefficient is strictly positive. When this fails globally, exact de Casteljau midpoint subdivision is used; see [1] for the Bernstein-basis background. All coefficients below are rational numbers.

Theorem 6.1 (All real harmonic cubics on $\mathbb{S}^2$)**.** *For every nonzero real degree-three spherical harmonic Y on $\mathbb{S}^2$, inequality* (9) *holds. Equivalently, if H is the homogeneous harmonic extension to $\mathbb{R}^3$, then*

$$\mathbb{E}\left[\left(H^2 - \frac{|DH|^2}{3}\right)\left(\frac{|DH|^2}{3}\right)^{3/20}\right] > 0.$$

Proof. Exact Bernstein conversion on the entire box (19) gives

$$\begin{aligned}
\min \mathcal{B}\left(\frac{114}{5}U_1 - V_1\right) &= \frac{4209}{1925} > 0,\\
\min \mathcal{B}(R_0) &= \frac{29152143969790092454188}{23165189182717957375} > 0,\\
\min \mathcal{B}(R_2) &= \frac{90692005649292}{445013694125} > 0.
\end{aligned}$$

The global control nets of R_1 and Δ contain negative entries, so neither polynomial is asserted to be globally positive from its initial net. Cyclic midpoint subdivision in (b, s, u) terminates with the following ten disjoint leaves:

$$\begin{array}{c|c c c|c}
LLLLL & \Delta & & LLLLR & R_1\\
LLLR & \Delta & & LLRLL & \Delta\\
LLRLR & R_1 & & LLRR & R_1\\
LRLL & R_1 & & LRLR & \Delta\\
LRR & R_1 & & R & \Delta
\end{array}$$

On each leaf, R_0 and R_2 have strictly positive control coefficients, and the displayed third polynomial has strictly positive control coefficients. The verifier prints the exact positive rational minimum on every leaf and asserts the complete leaf-path set. Hence (16) holds throughout (19).

The first positive endpoint and (17) imply $D_2 > 0$. Thus G is not constant under the probability measure proportional to $Y^2 d\sigma$. The matrix in (13) is the Gram matrix of 1 and G for the positive measure proportional to $G(G+t)Y^2 d\sigma$; hence it is positive definite for every $t \ge 0$, and clearing its determinant introduced no spurious sign. Therefore (14) and then (12) hold for every $t \ge 0$. The positive fractional integral representation proves (9). Proposition 2.1 gives the Gaussian form. $\square$

Remark 6.2 (What the computer verifies). The supplementary script uses SymPy only to manipulate rational polynomials and a NumPy object array whose entries are Python `Fraction` objects. It starts from (10), performs the sphere integrations using

$$\int_{\mathbb{S}^2} x^i y^j z^k \, d\sigma = \frac{(i-1)!!(j-1)!!(k-1)!!}{(i+j+k+1)!!}$$

for even i, j, k, and regenerates the complete ten-leaf tree. It contains no floating-point inequality. On a single CPU thread the full run takes about eight seconds on the authors' reference machine.

7 A nodal limitation of second-moment methods

The spherical form also identifies why a routine variance estimate cannot prove the general fractional inequality. Under probability measure proportional to $|Y|^q d\sigma$, define

$$U = M + \frac{|\nabla_{\mathbb{S}} Y|^2}{MY^2}.$$

Near a regular nodal point one may take a transverse coordinate y for which $|Y| \asymp |y|$ and $U \asymp |y|^{-2}$. Consequently

$$\mathbb{E}_{|Y|^q} U^a < \infty \quad \text{locally if and only if} \quad a < \frac{q+1}{2}. \tag{20}$$

At the resonant exponents considered here, $q = 2 + 2r < 3$. Hence $\mathbb{E}U^2 = \infty$, whereas the moment needed by the projective inequality is $a = r + 1 = q/2 < (q+1)/2$ and is finite. Thus Cauchy–Schwarz through an ordinary variance destroys exactly the fractional nodal information one needs. This observation is a limitation of that proof route, not evidence against the inequality.

8 Scope and open problem

Theorems 3.1, 4.1, and 6.1 prove complete low-degree and all-degree split families. They do not imply the same inequality for a linear combination of split tensors: the functional is nonlinear and cross terms survive. The natural remaining question is:

For every $M \geq 4$ and every nonzero homogeneous harmonic Gaussian chaos H of degree M, is $\mathcal{K}_{r_M}(H) > 0$ at the exponent (2)*?*

The result at $r = 1$ and the equality at $r = 0$ do not provide an interpolation theorem. Nor does the exact cubic certificate supply a dimension ladder. This boundary is important for the nonlinear cone application: the present paper closes the quadratic module and the first fully mixed cubic module, but it does not establish the all-degree projective gap or the independent nonlinear compactness/no-return statements needed for a full Kondrat'ev-type expansion.

A Reproduction protocol

The supplementary file `harmonic_cubic_phase_bernstein_exact.py` is run from a directory that does not shadow Python's standard library. A strictly single-threaded invocation is

```
OMP_NUM_THREADS=1 OPENBLAS_NUM_THREADS=1 MKL_NUM_THREADS=1 \
VECLIB_MAXIMUM_THREADS=1 NUMEXPR_NUM_THREADS=1 \
python3 harmonic_cubic_phase_bernstein_exact.py
```

The program asserts the invariant reconstruction, the factorization (17), the positivity of the global endpoint, R_0, and R_2 nets, the exact ten-leaf set, and maximum subdivision depth five. Any failed identity or incomplete leaf aborts the run.

Statements and declarations

Author contributions. K.K. conceived the project, developed the analytic arguments, and drafted the manuscript. D.K. contributed software implementation, exact-arithmetic verification, and review of the computational certificate. Both authors reviewed and approved the final manuscript.

Use of generative AI. Large-language-model tools (OpenAI Codex and Anthropic Claude) assisted exploratory derivations, source discovery and bibliographic cross-checking, generation and review of symbolic-verification code, and English-language editing. No AI system is listed as an author. The human authors reviewed the cited sources, mathematical arguments, code, and final text and take full responsibility for them.

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